$|V_{ub}|$ at Super-B

(Inclusive $B \to X_s\gamma$ and $B \to X_u\ell\bar{\nu}$)

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KEK, March 17–19, 2009

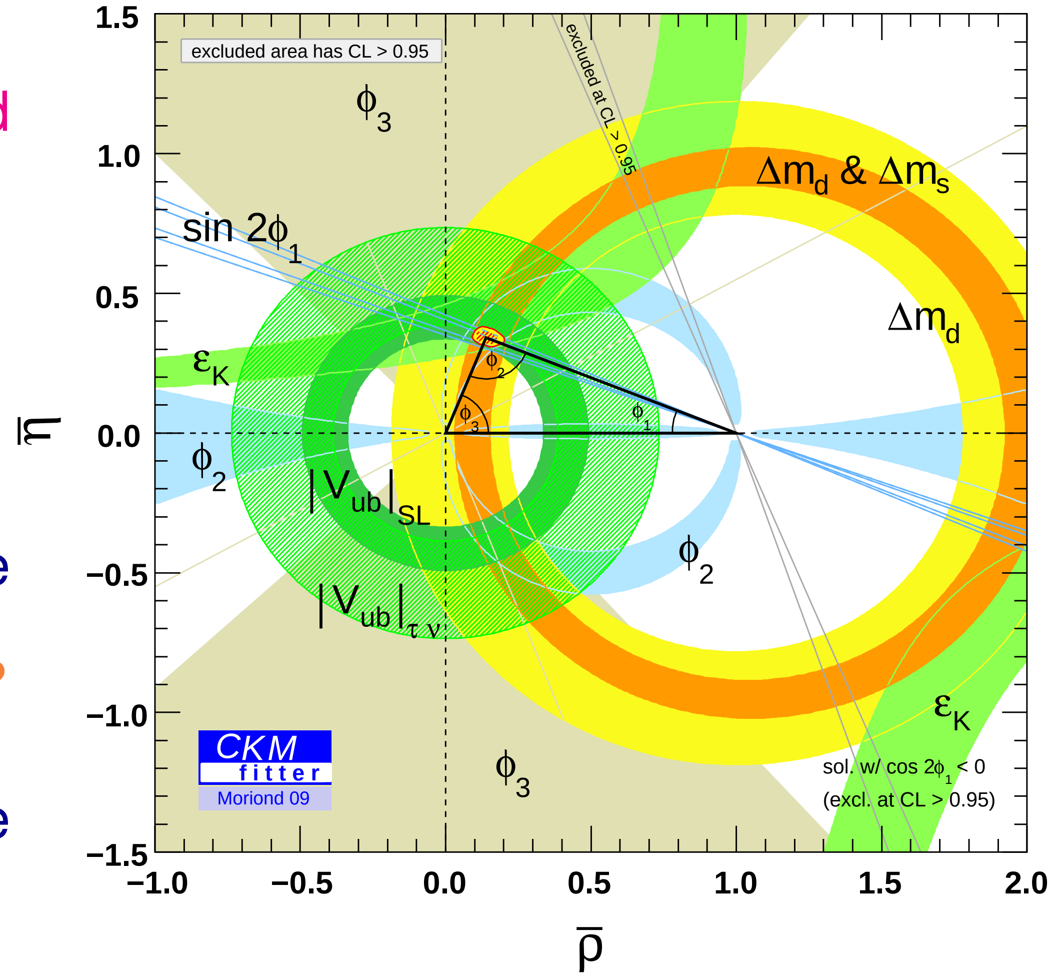
The importance of $|V_{ub}|$

- $|V_{ub}|$ determined by tree-level decays
  Crucial for comparing tree-dominated and loop-mediated processes

- $|V_{ub}|_{\pi\ell\bar{\nu}-\mathrm{LQCD}} = (3.5 \pm 0.5) \times 10^{-3}$
  $|V_{ub}|_{\mathrm{incl-BLNP}} = (4.32 \pm 0.35) \times 10^{-3}$
  $|V_{ub}|_{\tau\nu} = (5.2 \pm 0.5 \pm 0.4_{f_B}) \times 10^{-3}$
  SM CKM fit, $\sin 2\phi_1$, favors small value

- Fluctuation, bad theory, new physics?

- The level of agreement between the measurements often misinterpreted



- The question has been who sees NP first; once it's seen, how to understand it?

Main reason (for me) to continue

- Overconstraining ("redundant") measurements are crucial to bound new physics

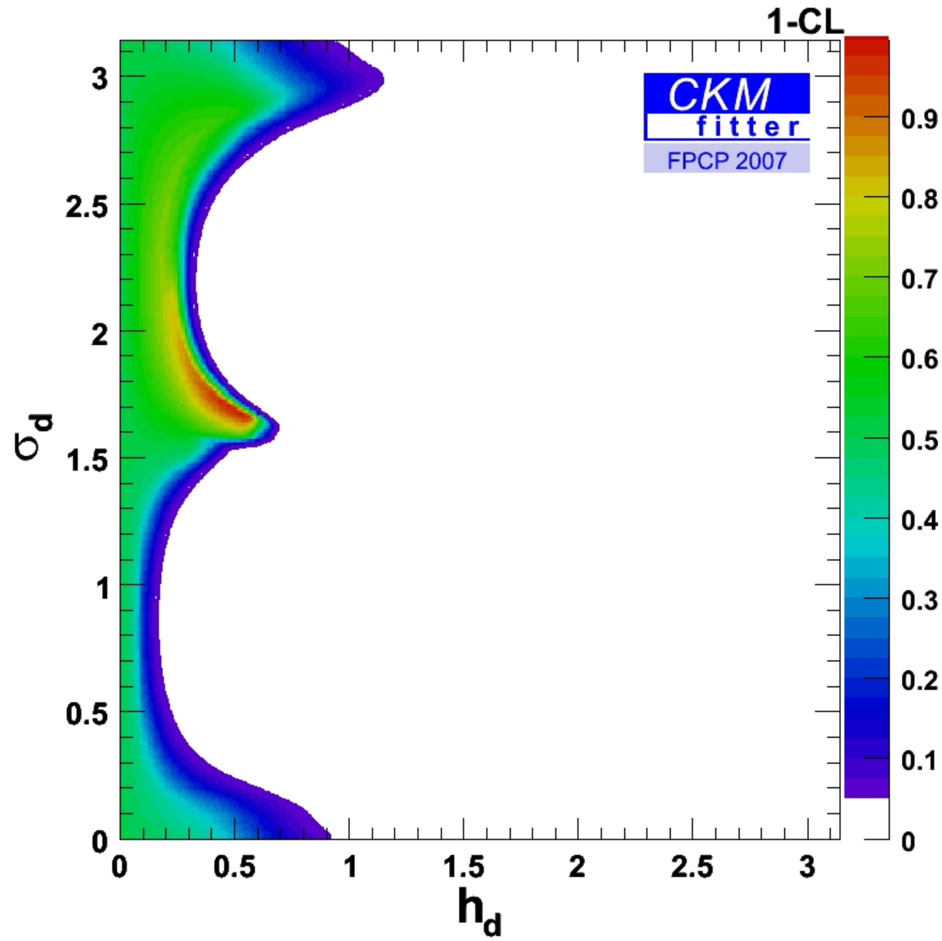
  Parameterization of NP in B^0–$\overline{B}^0$ mixing:

  $$M_{12} = M_{12}^{\rm SM}\,(1 + h_d\, e^{2i\sigma_d})$$

- Non-SM terms not yet bound to be $\ll$ SM

  What we really ask: is $\Lambda_{\rm flavor} \gg \Lambda_{\rm EWSB}$?

  Need lot more data to determine whether NP $\ll$ SM (unless $\sigma_d = 0 \bmod \pi/2$)



- 10–20% non-SM contributions to most loop-mediated transitions are still possible

  $\Rightarrow$ In my mind building a Super-B-factory is clearly justified!

Outline

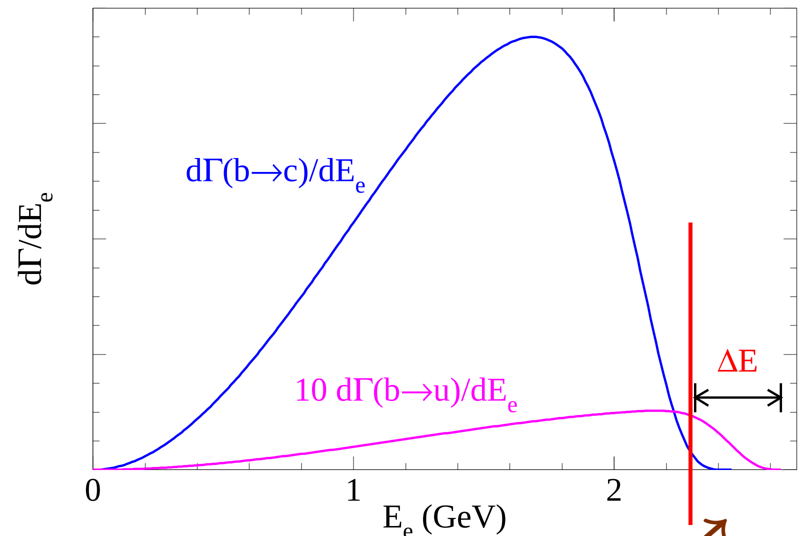
- Introduction to past inclusive analyses
- Complete description of $B \to X_s\gamma$ [ZL, Stewart, Tackmann, arXiv:0807.1926]
- Complete description of $B \to X_u\ell\bar{\nu}$ [ZL, Stewart, Tackmann, to appear]
- A glimpse at SIMBA [Bernlochner, Lacker, ZL, Stewart, Tackmann, Tackmann, to appear]
- Outlook

The challenge of inclusive $|V_{ub}|$ measurements

- Total rate known with $\sim 4\%$ accuracy, similar to $\mathcal{B}(B \to X_c \ell \bar{\nu})$ [Hoang, ZL, Manohar]

- To remove the huge charm background ($|V_{cb}/V_{ub}|^2 \sim 100$), need phase space cuts

  Phase space cuts can enhance perturbative and nonperturbative corrections drastically



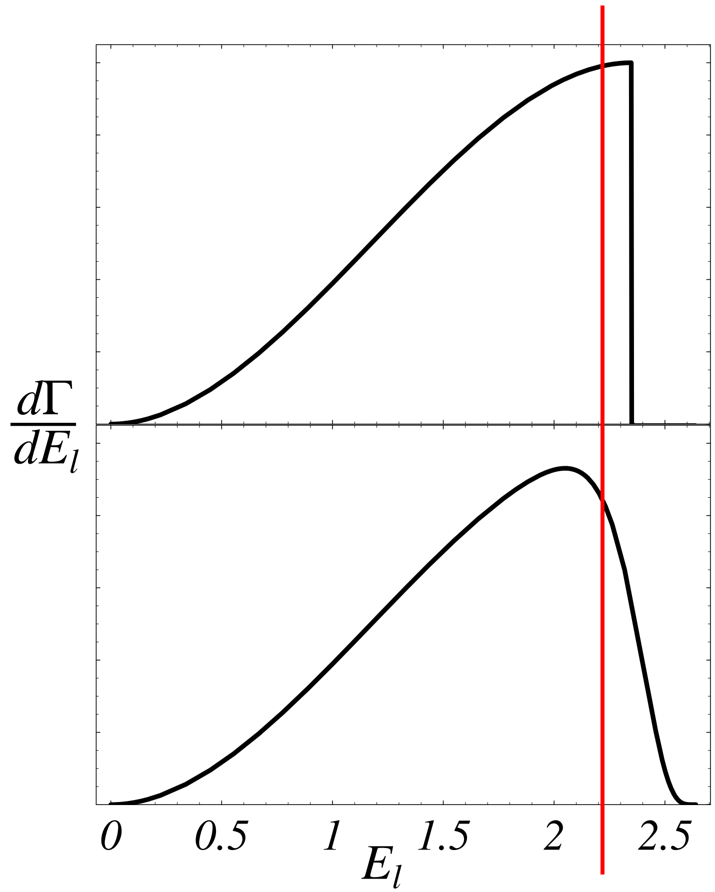
  Nonperturbative effects shift endpoint $\frac{1}{2} m_b \to \frac{1}{2} m_B$ and determine shape

- Endpoint region determined by b quark PDF in B; want to extract it from data to make predictions — at lowest order $\propto B \to X_s \gamma$ photon spectrum [known since '94]

$|V_{ub}|$: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

with a model for b quark PDF



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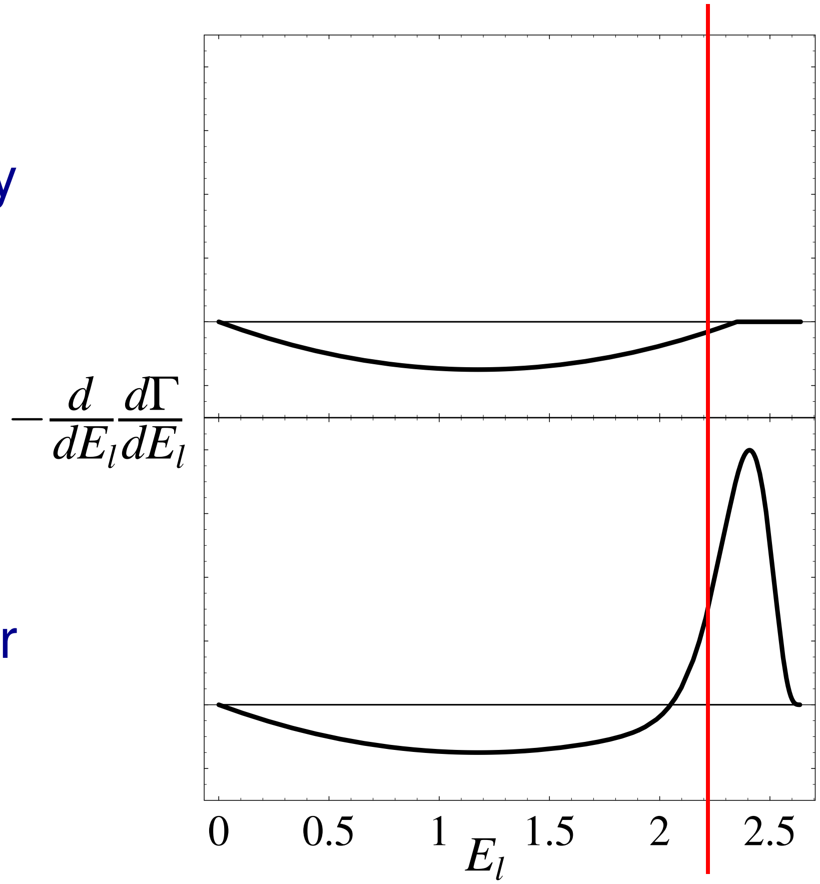

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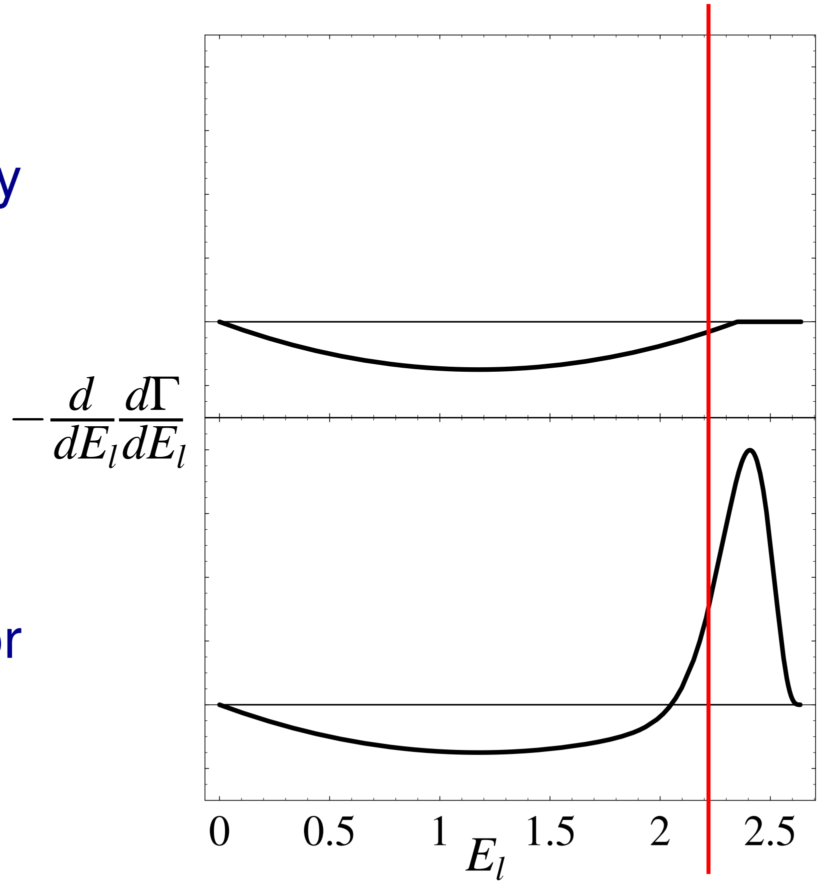
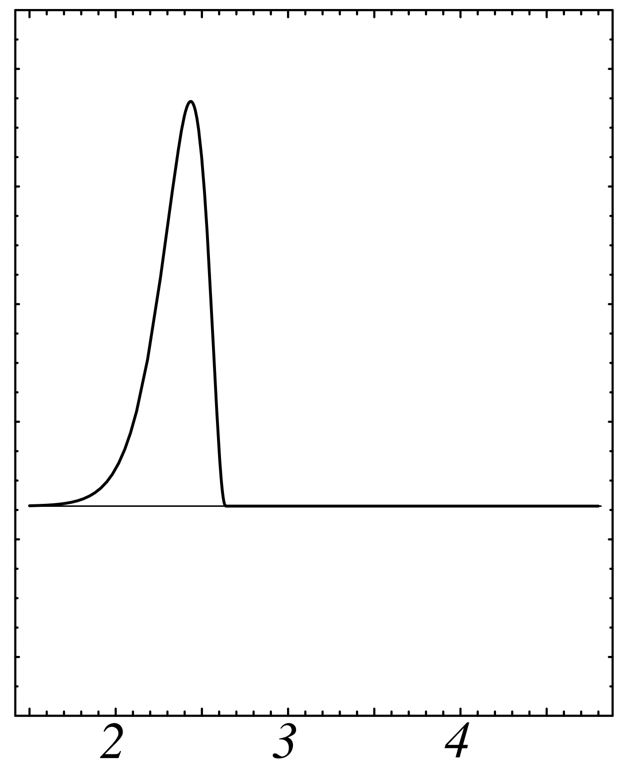


with a model for b quark PDF

difference:



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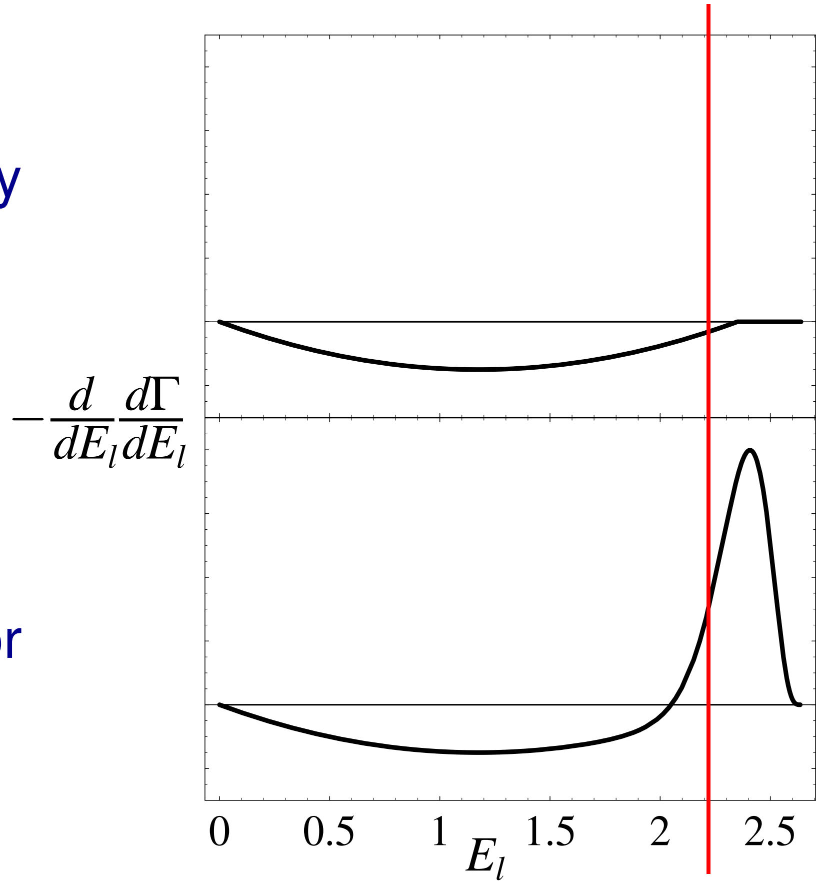

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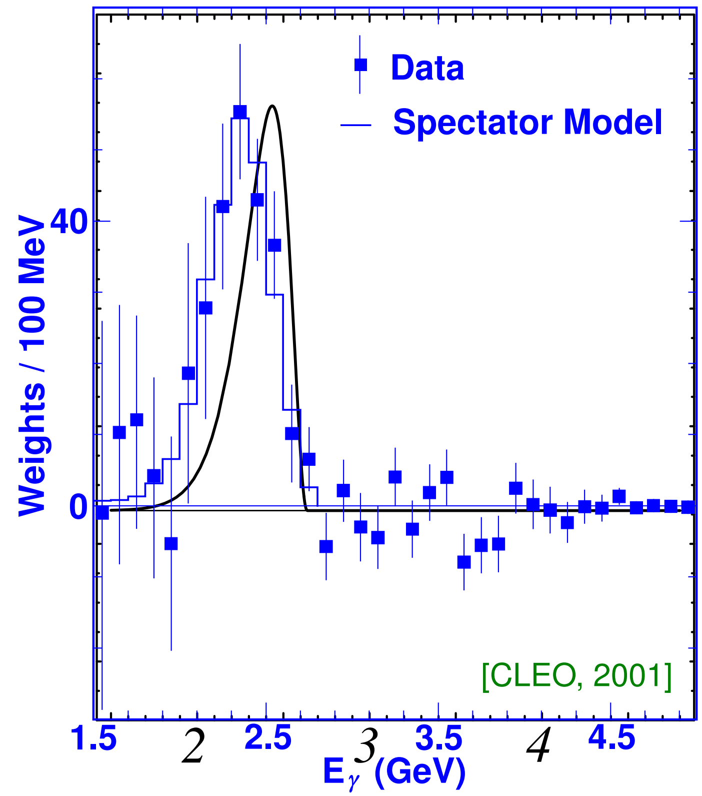

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b quark decay spectrum

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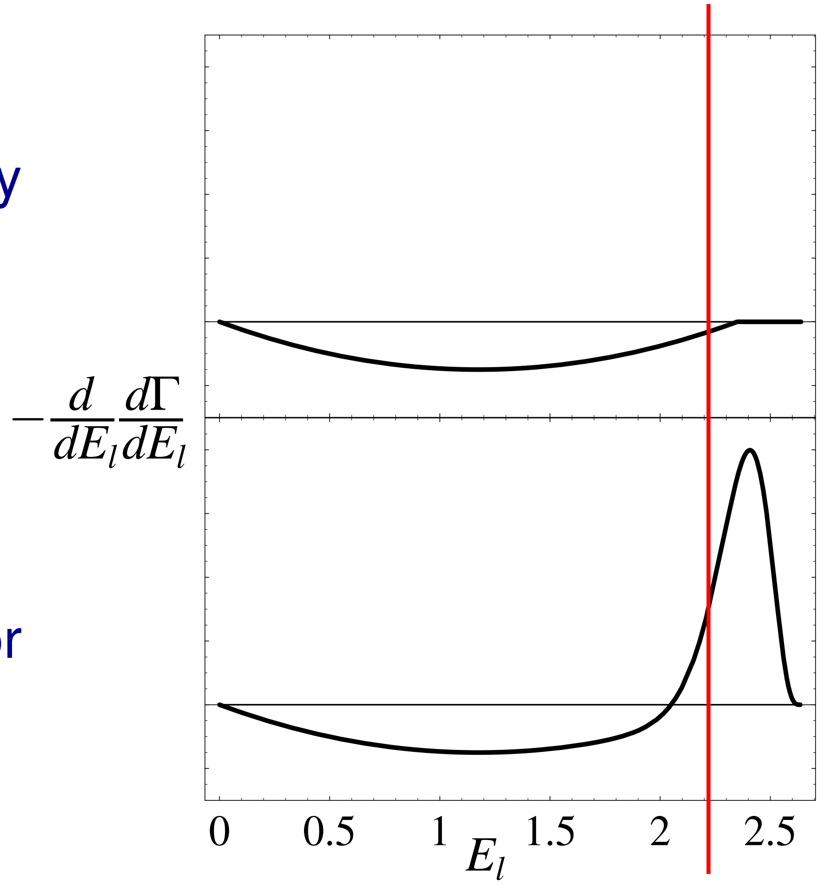

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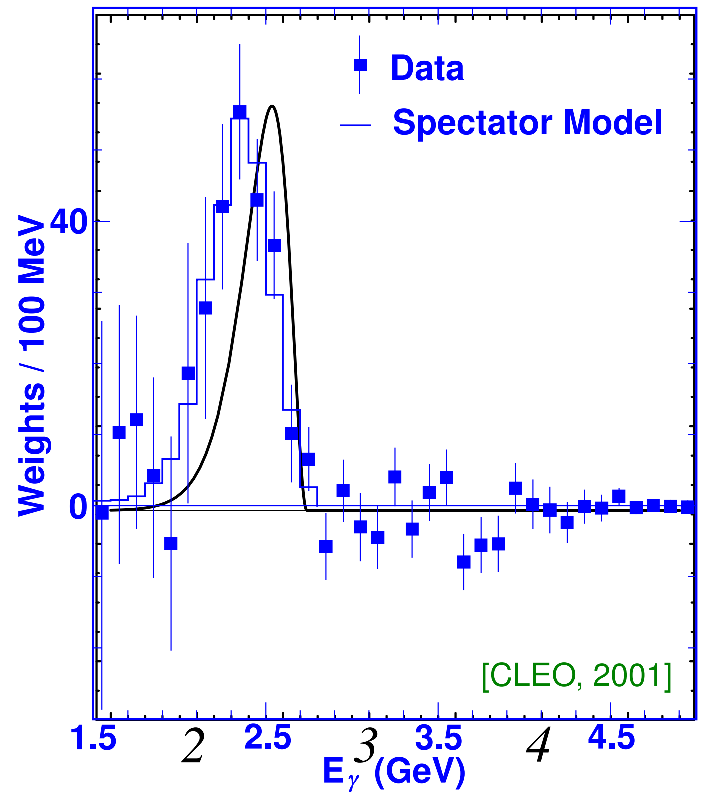


- Both of these spectra are determined at lowest order by the b quark PDF in the B
- Was no fully consistent formalism beyond lowest order, without ad hoc ingredients

Past efforts: BLNP (best so far)

- Treated factorization & resummation in shape function region correctly
- Use specific (ad-hoc) functional forms to model shape function
- Shape function scheme for m_b, λ_1 (One scheme for each approach)
- Awkward "tail gluing" to make shape function's moments consistent with RGE (not even done in approaches other than BLNP)

$\Rightarrow$ Hard to assess uncertainties

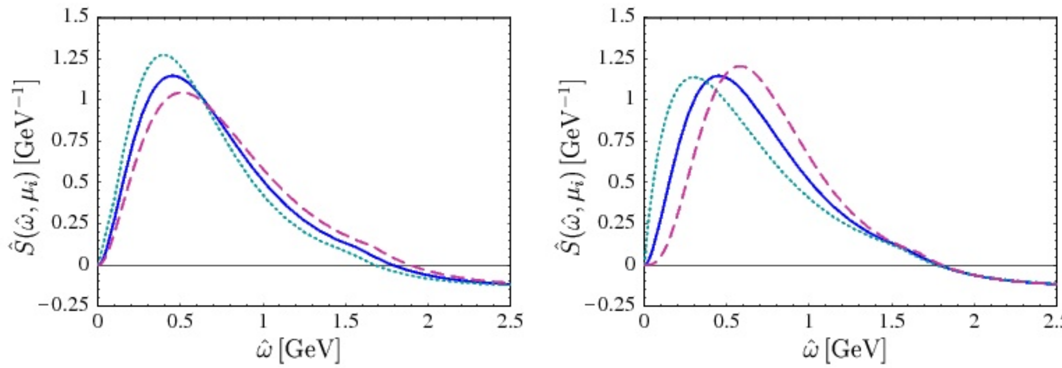


Figure 6: Various models for the shape function at the intermediate scale $\mu_i = 1.5\,\mathrm{GeV}$, corresponding to different parameter settings in Table 1. Left: Functions S1, S5, S9 with "correlated" parameter variations. Right: Functions S3, S5, S7 with "anti-correlated" parameter variations.

[Bosch, Lange, Neubert, Paz]

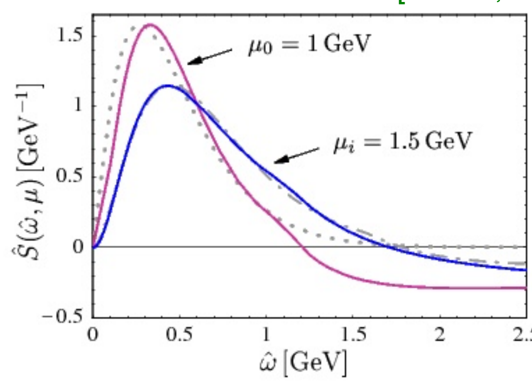


Figure 7: Renormalization-group evolution of a model shape function from a low scale μ_0 (sharply peaked solid curve) to the intermediate scale μ_i (broad solid curve). See the text for an explanation of the other curves.

Start with $B \to X_s\gamma$

[ZL, Stewart, Tackmann, PRD 78 (2008) 114014, arXiv:0807.1926]

Regions of phase space

- $B \to X_s\gamma$ gives one of the strongest bounds on NP
- $m_B - 2E_\gamma \lesssim 2\,\text{GeV}$, and $< 1\,\text{GeV}$ at the peak

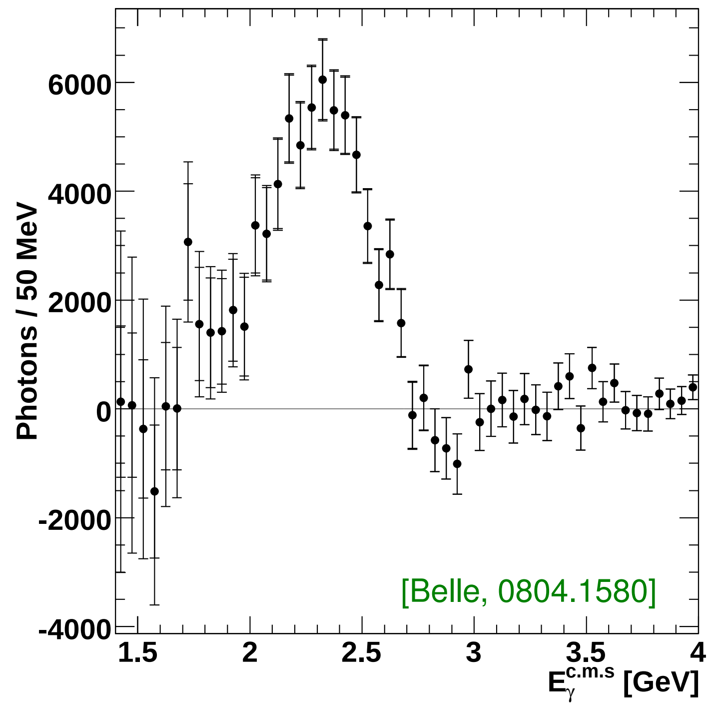
Three cases: 1) $\Lambda \sim m_B - 2E_\gamma \ll m_B$

2) $\Lambda \ll m_B - 2E_\gamma \ll m_B$

3) $\Lambda \ll m_B - 2E_\gamma \sim m_B$

Neither 1) nor 2) is fully appropriate

Can combine 1) – 2) w/o expanding $\Lambda/(m_B - 2E_\gamma)$



- Include all known results in regions 1) – 2) (sometimes called SCET and MSOPE)

LL: 1–loop Γ_{cusp}, tree-level matching

NLL: 2–loop Γ_{cusp}, 1–loop matching, 1–loop γ_x

NNLL: 3–loop Γ_{cusp}, 2–loop matching, 2–loop γ_x

The shape function (b quark PDF in B)

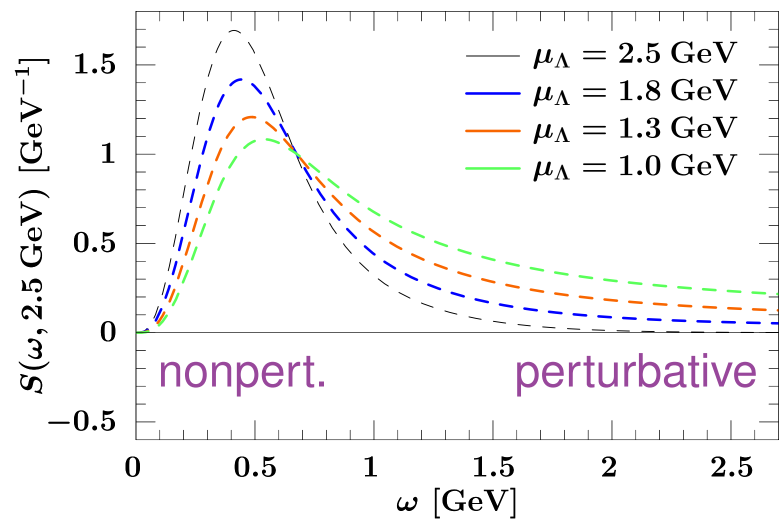
- The shape function $S(\omega, \mu)$ contains nonperturbative physics and obeys a RGE

If $S(\omega, \mu_\Lambda)$ has exponentially small tail, any small running gives a long tail and divergent moments

$$S(\omega, \mu_i) = \int \mathrm{d}\omega' \, U_S(\omega - \omega', \mu_i, \mu_\Lambda) \, S(\omega', \mu_\Lambda)$$

Constraint: moments (OPE) + $B \to X_s \gamma$ shape

How to combine these?



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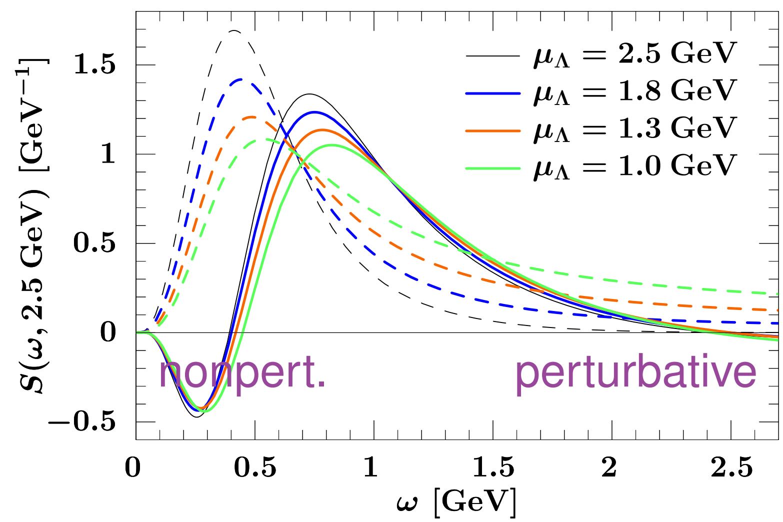
Constraint: moments (OPE) + $B \to X_s\gamma$ shape

How to combine these?

Derive: [ZL, Stewart, Tackmann, 0807.1926]

$$S(\omega,\mu_\Lambda) = \int \mathrm{d}k\, C_0(\omega-k,\mu_\Lambda) F(k)$$



Model $\begin{cases} S & \text{(dash)} \\ F & \text{(solid)} \end{cases}$ run to $2.5\mathrm{GeV}$

– Consistent setup at any order, in any scheme

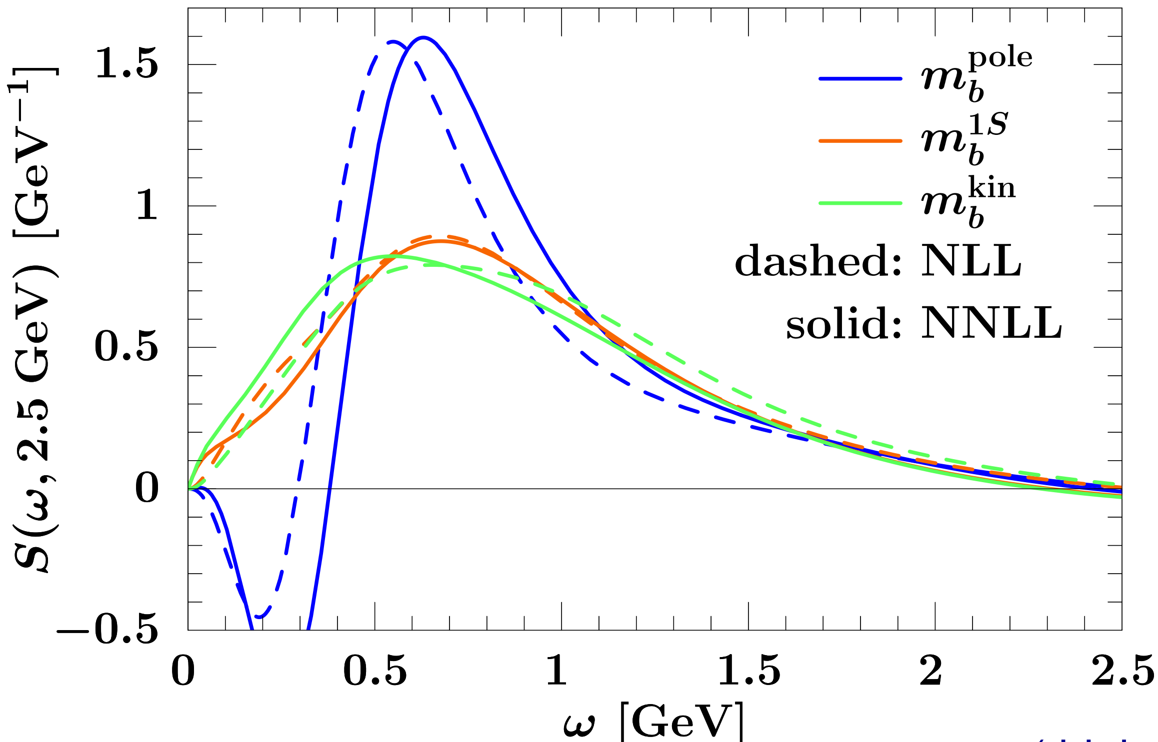
– Stable results for varying μ_Λ (SF modeling scale, must be part of uncert.)

– Analog of how all matrix elements are defined

- Consistent to impose moment constraints on $F(k)$, but not on $S(\omega,\mu_\Lambda)$ w/o cutoff

Schemes for m_b

- Converting results to a short distance mass scheme removes dip at small ω:

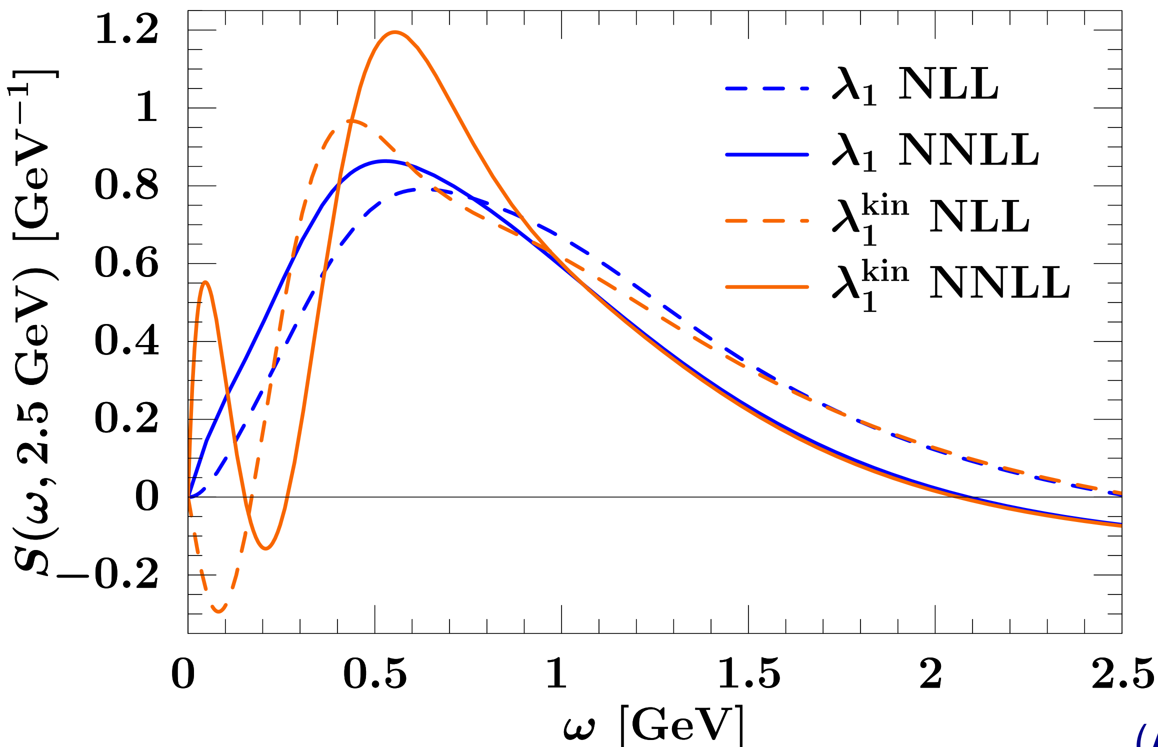


(think of ω as $m_B - 2E_\gamma$)

- Can use any short distance mass scheme ($1S$, kinetic, PS, shape function, ...)

Schemes for λ_1

- We find that kinetic scheme, $\mu_\pi^2 \equiv -\lambda_1^{\text{kin}}$, oversubtracts; similar to $\overline{m}_b(\overline{m}_b)$ issues

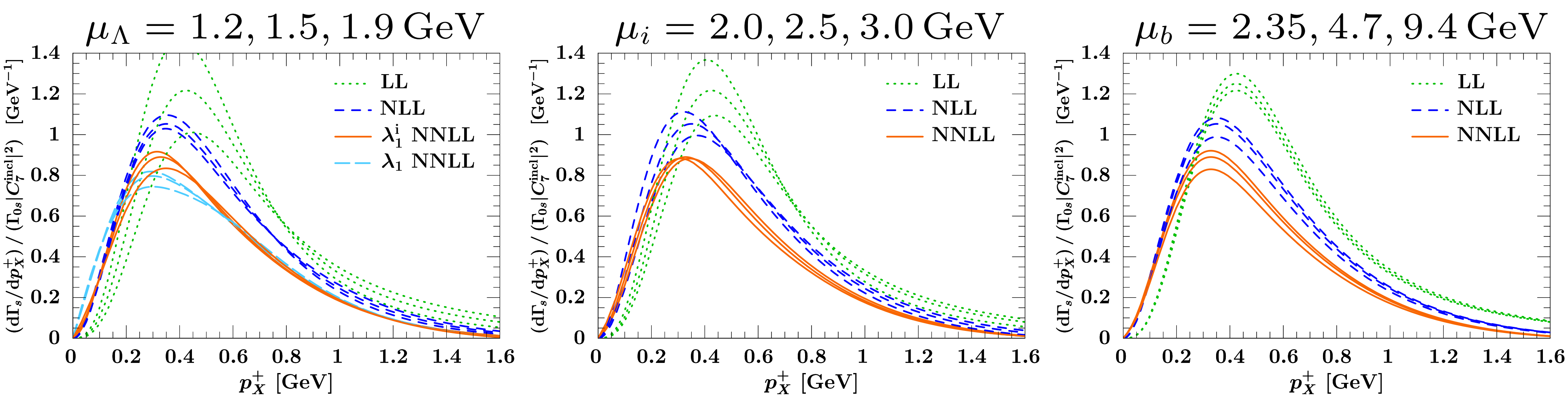


(All curves use m_b^{kin})

- Introduce "invisible" scheme: $\lambda_1^{\text{i}} = \lambda_1 - 0\alpha_s - R^2\,\frac{\alpha_s^2(\mu)}{\pi^2}\,\frac{C_F C_A}{4}\left(\frac{\pi^2}{3} - 1\right)$ $\quad(R = 1\,\text{GeV})$

Scale (in)dependence of $B \to X_s\gamma$ spectrum

- Dependence on 3 scales in the problem:



$$\frac{\mathrm{d}\Gamma_s}{\mathrm{d}p_X^+} = \Gamma_{0s}\, H_s(p_X^+, \mu_b)\, U_H(m_b, \mu_b, \mu_i) \int \mathrm{d}k\, \widehat{P}(m_b, k, \mu_i)\, \widehat{F}(p_X^+ - k) \qquad \left(p_X^+ = m_B - 2E_\gamma\right)$$

$\widehat{P}, \widehat{F}$ indicate use of short distance schemes: m_b^{1S} and λ_1^{i}

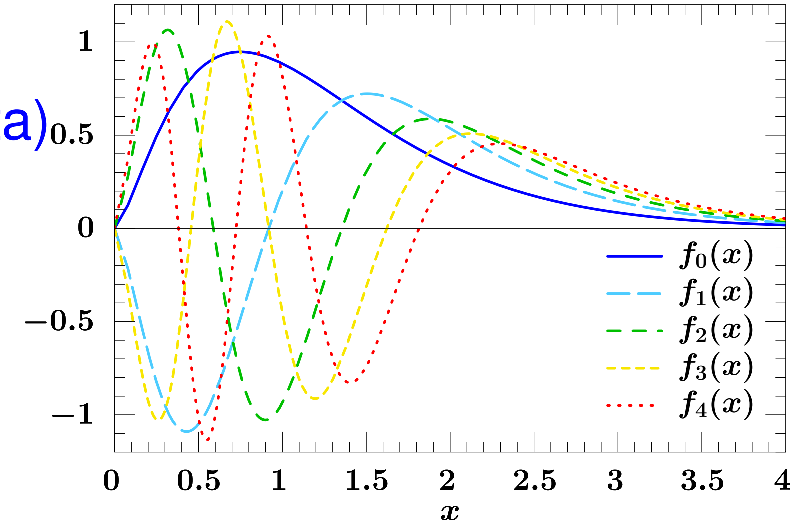
- In other approaches, using models for $S(\omega, \mu_\Lambda)$ run up to μ_i, dependence on μ_Λ ignored so far, but it must be considered an uncertainty $\Rightarrow$ This is how to solve it

Designer orthonormal functions

- Devise suitable orthonormal basis functions (earlier: fit parameters of model functions to data)

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\Big[\sum c_n f_n(x)\Big]^2,\ n \text{ th moment} \propto \Lambda_{\rm QCD}^n$$

$$f_n(x) \sim P_n[y(x)] \leftarrow \text{Legendre polynomials}$$

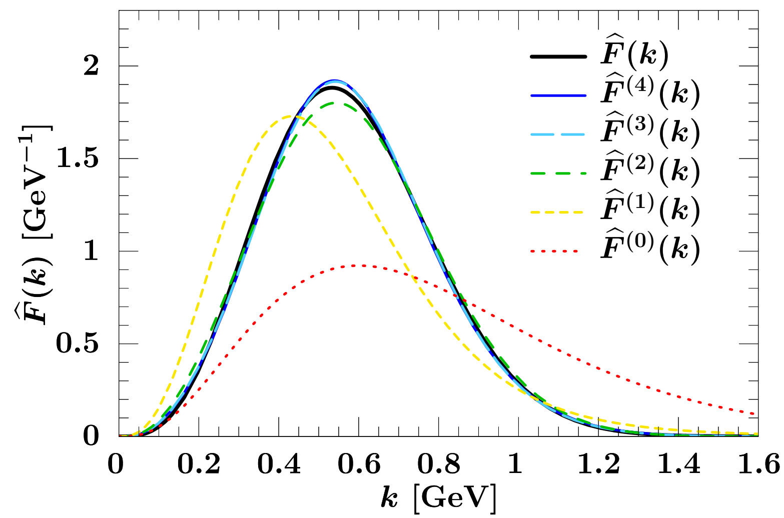


- Approximating a model shape function

Better to add a new term in an orthonormal basis than a new parameter to a model:

– less parameter correlations

– errors easier to quantify



"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." (John von Neumann)

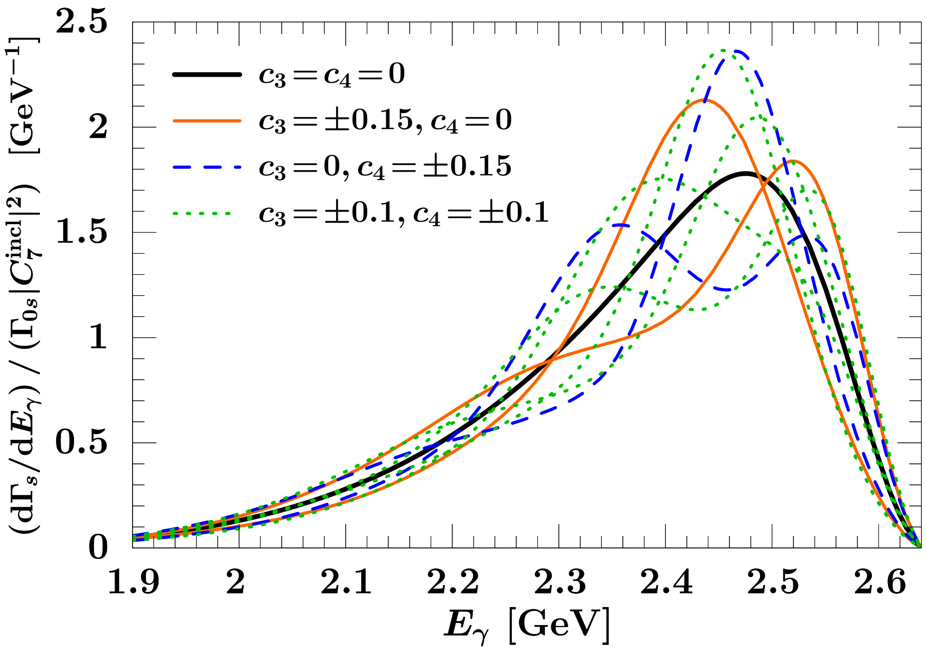
Back to the $B \to X_s\gamma$ spectrum

- 9 models w/ same 0th, 1st, 2nd moments

  Including all NNLL contributions, find:

  – Shape in peak region not determined at all by first few moments

  – Smaller shape function uncertainty for $E_\gamma \lesssim 2.1\,\mathrm{GeV}$ than earlier studies



- Neglected in this plot: subleading shape functions
  subleading corrections not in $C_7^{\mathrm{incl}}(0)$
  boost to $\Upsilon(4S)$ frame

- These results can also be used for: $B \to X_s \ell^+\ell^-$, $|V_{ub}|$

More complicated: $B \to X_u \ell \bar{\nu}$

[ZL, Stewart, Tackmann, to appear]

Regions of phase space (again)

- "Natural" kinematic variables: $p_X^\pm = E_X \mp |\vec{p}_X|$ — "jettyness" of hadronic final state

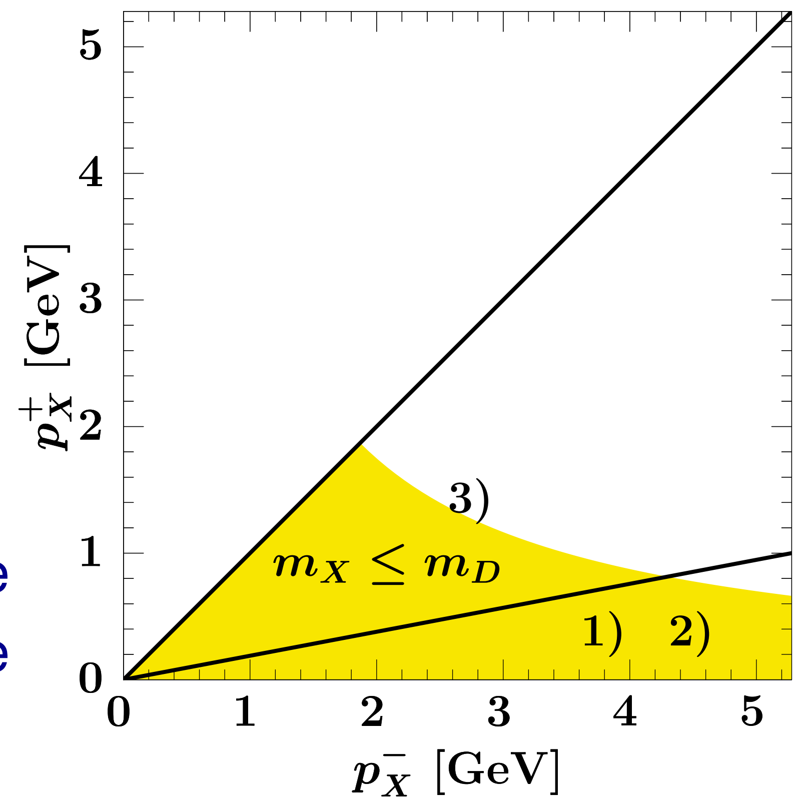
  $B \to X_s\gamma$: $p_X^+ = m_B - 2E_\gamma$ & $p_X^- \equiv m_B$, but independent variables in $B \to X_u \ell\bar\nu$

  Three cases: 1) $\Lambda \sim p_X^+ \ll p_X^-$

  2) $\Lambda \ll p_X^+ \ll p_X^-$

  3) $\Lambda \ll p_X^+ \sim p_X^-$

  Make no assumptions how p_X^- compares to m_B



- $B \to X_s\gamma$: small rate in region 3), not important

- $B \to X_u \ell\bar\nu$: 3-body final state, appreciable rate in region 3), where hadronic final state not jet-like

  E.g., $m_X^2 < m_D^2$ does not imply $p_X^+ \ll p_X^-$

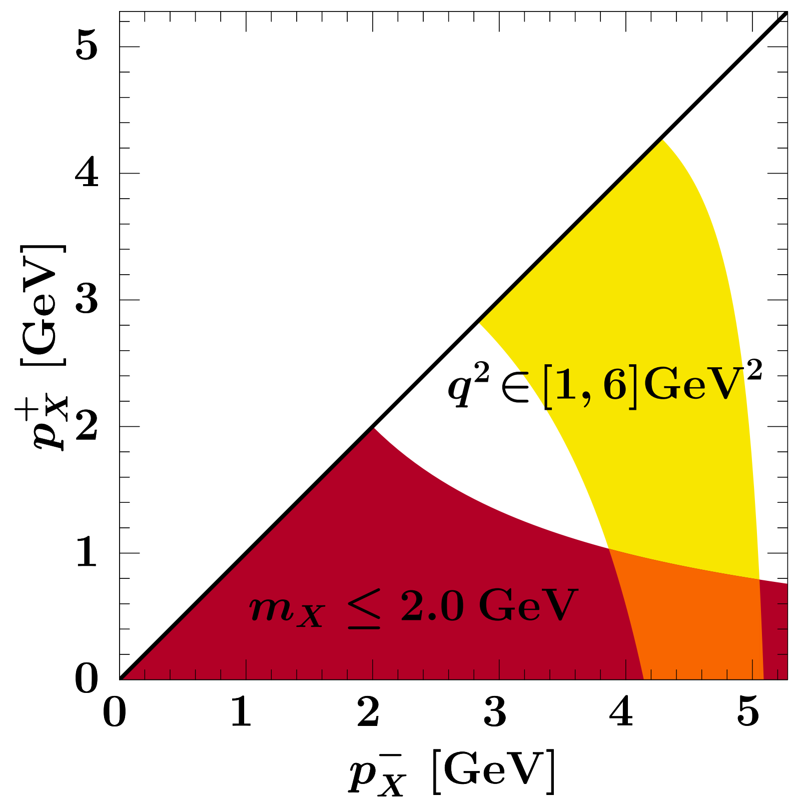
- Can combine 1)$-$2) just as we did in $B \to X_s\gamma$, by not expanding in Λ/p_X^+

Regions of phase space (aside)

- Main difference in $B \to X_s \ell^+ \ell^-$: only regions 1) – 2) are relevant for $q^2 < m_\psi^2$

  The $q^2 > m_{\psi'}^2$ region is like large-q^2 in $B \to X_u \ell \bar{\nu}$

(More about this tomorrow)



Charmless $B \to X_u \ell \bar{\nu}$ made charming

- To combine all 3 regions: do not expand in Λ/p_X^+ nor in p_X^+/p_X^-
- Want to have: result accurate to NNLL and $\Lambda_{\rm QCD}/m_b$ in regions 1)–2) and to order $\alpha_s^2\beta_0$ and $\Lambda_{\rm QCD}^2/m_b^2$ when phase space limits are in region 3)
- Hopefully we'll soon have a tool which can be used both to evaluate measurements and as a generator

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Start with triple differential rate (involves a delta-fn at the parton level at $\mathcal{O}(\alpha_s^0)$, which is smeared by the shape function)

The p_X^+/p_X^- terms, which are not suppressed in local OPE region, start at $\mathcal{O}(\alpha_s)$

Recently completed $\mathcal{O}(\alpha_s^2)$ matching computations contains the needed results

[Bonciani & Ferroglia, 0809.4687; Asatrian, Greub, Pecjak, 0810.0987; Beneke, Huber, Li, 0810.1230; Bell, 0810.5695]

SIMBA

SIMBA – Analysis of B-Meson Inclusive Spectra –

[Bernlochner, Lacker, ZL, Stewart, Tackmann, Tackmann, to appear]

Fitting charmless inclusive decay spectra

- Fit strategy: $\widehat{F}(k)$ enters the spectra linearly $\Rightarrow$ can calculate independently the contribution of $f_m\, f_n$ in the expansion of $\widehat{F}(k)$:

$$\mathrm{d}\Gamma = \sum \underbrace{c_m\, c_n}_{\text{fit}}\; \underbrace{\mathrm{d}\Gamma_{mn}}_{\text{compute}}$$

$$\mathrm{d}\Gamma_{mn} = \Gamma_0\, H(p^-) \int_0^{p_X^+} \mathrm{d}k\, \frac{\widehat{P}(p^-, k)}{\lambda}\, \underbrace{f_m\Big(\frac{p_X^+ - k}{\lambda}\Big)\, f_n\Big(\frac{p_X^+ - k}{\lambda}\Big)}_{\text{basis functions}}$$

  Fit the c_i coefficients from the measured (binned) spectra

- What we hope to achieve:
  - Correlation and error propagation of SF uncertainties
  - Simultaneous fit using all available information
  - Can add or remove parameters to estimate model uncertainties
  - Reduced correlations between model parameters (orthonormal basis)

A preliminary $B \to X_s\gamma$ fit

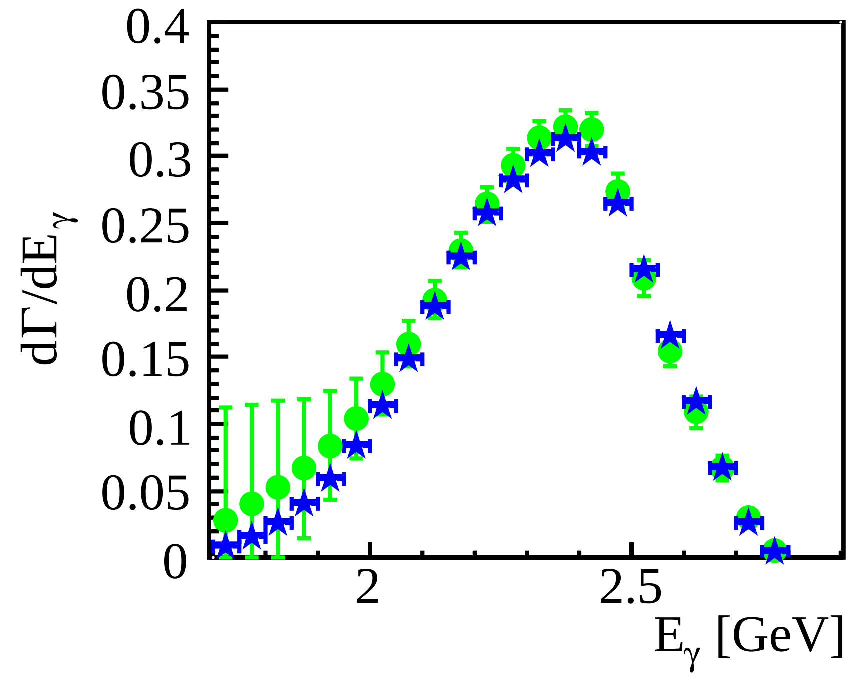
- Belle $B \to X_s\gamma$ spectrum in $\Upsilon(4S)$ restframe

  For demonstration purposes only — there are very strong correlations

  Fit with 4 basis functions in the expansion of the shape function

  Shows that fit works (not as trivial as this plot might indicate); still issues to resolve

[Belle, 0804.1580 + Preliminary; thanks to Antonio Limosani]



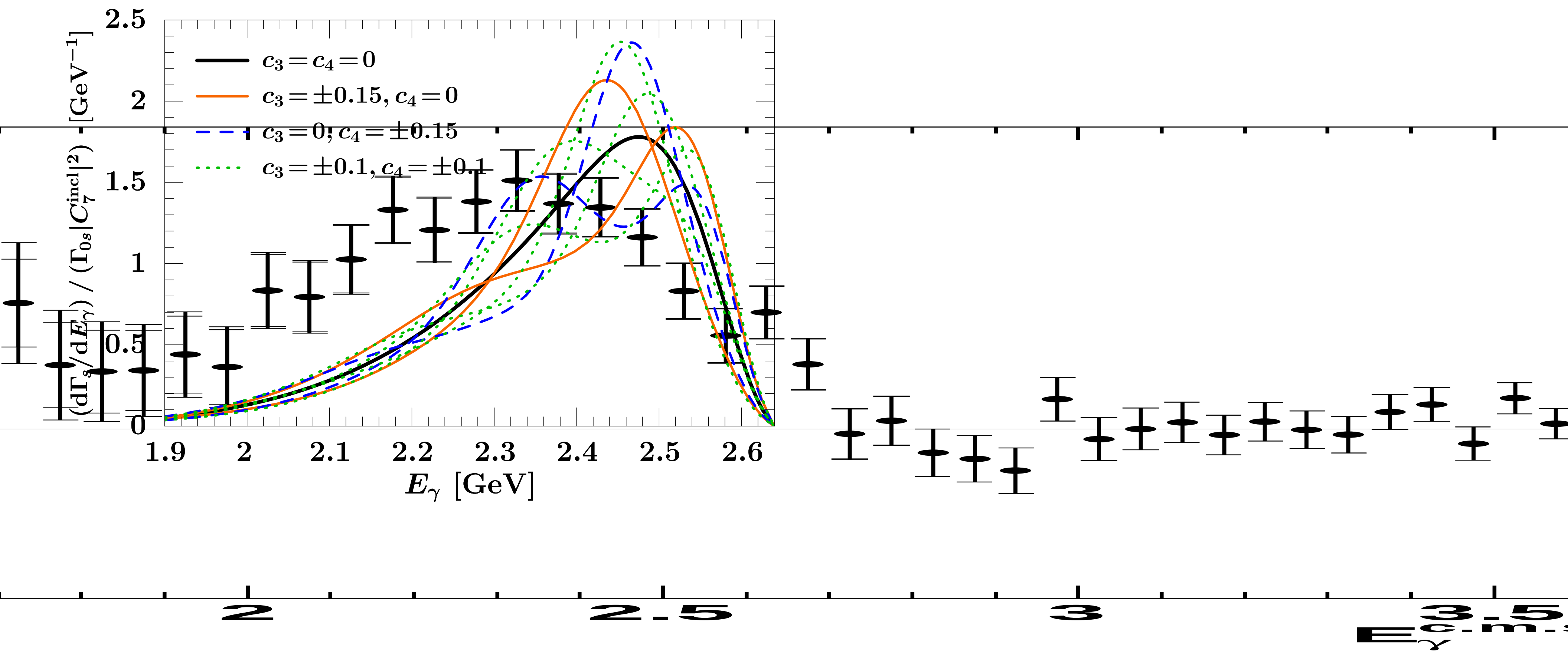
- Next step: include various $B \to X_u \ell \bar{\nu}$ measurements

Conclusions

- Improving accuracy of $|V_{ub}|$ will remain important to constrain new physics (Current situation unsettled, PDG in 2008 inflated error for the first time)
- Qualitatively better inclusive analyses possible than those implemented so far Developments will allow combining all pieces of data with tractable uncertainties
- To draw conclusions about new physics comparing sides and angles, we'll want ≥ 2 extractions of $|V_{ub}|$ with different uncertainties (inclusive, exclusive, leptonic)
- Don't give up inclusive... ability to do B reconstruction... hermeticity... important also for $B \to X_s \ell^+ \ell^-$, final states with τ's and ν's, etc.

Backup slides

The c.m. frame $B \to X_s\gamma$ spectrum



Derivation of shape function formula

- The shape function is the B meson matrix element of a nonlocal operator:

$$S(\omega,\mu) = \langle B|\, \underbrace{\bar b_v\, \delta(iD_+ - \delta + \omega)\, b_v}_{O_0(\omega,\mu)} \,|B\rangle, \qquad \delta = m_B - m_b$$

Integrated over a large enough region, $0 \le \omega \le \Lambda$, one can expand O_0 as

$$O_0(\omega,\mu) = \sum C_n(\omega,\mu)\ \underbrace{\bar b_v\, (iD_+ - \delta)^n\, b_v}_{Q_n} + \ldots = \sum C_n(\omega-\delta,\mu)\ \underbrace{\bar b_v\, (iD_+)^n\, b_v}_{\widetilde Q_n} + \ldots$$

C_n same for Q_n and $\widetilde Q_n$ (since O_0 only depends on $\omega-\delta$), determined by matching

Evaluating $\langle O_0\rangle_{b_v(k)}$, can show using RPI: $C_n(\omega,\mu) = \dfrac{1}{n!}\, \dfrac{\mathrm{d}^n C_0(\omega,\mu)}{\mathrm{d}\omega^n}$ [Bauer & Manohar]

- Define the nonperturbative function $F(k)$ by:

$$S(\omega,\mu_\Lambda) = \int \mathrm{d}k\, C_0(\omega-k,\mu_\Lambda)\, F(k), \qquad C_0(\omega,\mu) = \langle b_v|\, O_0(\omega+\delta,\mu)\, |b_v\rangle$$

Expand in k, compare: $\int \mathrm{d}k\, k^n F(k) = (-1)^n \langle Q_n\rangle_B$, so fully consistent with OPE